Analytical and Numerical Test of ISS Internal Multiple Attenuator on Data with Q Influence

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Abstract

ISS (Inverse Scattering Series) internal multiple attenuating algorithm can predict internal multiple with exact phase and approximate amplitude with spike data. However, the wave is decaying and broadening if propagating in attenuating medium, which is expressed by Quality Factor (Q). In this paper, the ISS internal multiple attenuator is analytically and numerically tested with absorptive data. The results show that if the events are isolated, the predicted multiple has a right phase and approximate amplitude which worse than that with ideal input data.

1 Introduction

ISS internal multiple attenuation algorithm will have a significant effect if the events in the data are sharpened. In order to improve the result, preprocessing works, including wavefield separation and deconvolution, can not be ignored. However, in the absorptive earth, the wavefield will be decayed and broadened even with preprocessing. Q compensation based on ISS show its satisfying effectiveness (see e.g. Innanen and Weglein (2003), Innanen and Weglein (2005), Innanen and Lira (2008)). For data with Q absorption, ISS internal multiple attenuator is first tested in this paper. A two-reflector model with constant Q in each layer is used for analytical and numerical verification. The result indicate that the prediction is with a right phase and an approximate amplitude if events are isolated; however, comparing with the data without Q, the amplitude is worth for the former case.

2 Q definition and wavefield representation

2.1 Q definition

Based on Aki and Richards (2002), the Q is used to represent the energy lost for wave filed propagating in one wave length. Q can be defined as

(1)

where E is the energy of wave field, and ΔE is the energy lost in one wavelength propagation. With the definition of Q, the amplitude of wave field A along propagation direction x can be represented as

(2)

where is the amplitude without absorption influence. ω is the frequency, and c is the velocity of the wave field. The exponentially decaying term causes the attenuation and results in a wave with finite width, rather than the original spike. It's not difficult to understand when Q decreases, the amplitude will decrease; otherwise, when Q increases to infinity, there's no absorption influence.

Here we suppose Q is frequency independent. In order to guarantee the amplitude attenuates in the negative frequency, it's convenient to replace ω with , then

(3)

2.2 Wavefield representation with layered model

Following the definition in the previous section, we can analytically express the wavefield. Take a two-half space 1D model with an interface at depth as an example (Figure 1), consider there are different velocities c, densities (ϱ) and Q values above and below the interface. The source is located at depth , above the interface.

Figure 1: A two half space model with an interface at depth

The 1D normal incident plane waves can be represented as:

[Sorry. Ignored \begin{aligned} ... \end{aligned}]

) (4)

where and are wavefields above and below the interface separately, R is reflection coefficient and T is the transmission coefficient. is composed by direct wave and reflected wave, both of which have exponential decaying terms; similarly, is the transmitted wave under Q influence.

If we define the wavenumber as

[Sorry. Ignored \begin{aligned} ... \end{aligned}]

) (5)

Then the wavefieds can be rewritten as

[Sorry. Ignored \begin{aligned} ... \end{aligned}]

) (6)

which are in the same form as the wavefileds expression without absorption, except the wavenumbers here are complex values.

By using the boundary conditions that pressures and displacements are continuous along the boundary, the reflection and transmission coefficients can be determined.

[Sorry. Ignored \begin{aligned} ... \end{aligned}]

) (7)

R and T here are both complex values.

3 Analytical test of ISS internal multiple attenuation algorithm on data with Q

In this section, I will use the attenuated data as input to test the ISS internal multiple attenuation algorithm analytically. Take a two-reflector model as an example, the parameters are listed on Figure 2, and both the depths of source and receiver are assumed to be 0.

Figure 2: A two-reflector model. and are primaries from the first and second interface separately.

For 1D model and the 1D normal incident plane wave, two primaries in the data can be expressed as:

[Sorry. Ignored \begin{aligned} ... \end{aligned}]

)   (8)

The migrated data in pseudo depth domain will be input to the internal multiple attenuation integral algorithm. First, the variable should be changed from ω to , then

[Sorry. Ignored \begin{aligned} ... \end{aligned}]

)   (9)

Then, Fourier Transform is applied over to pseudo depth domain. We can get

[Sorry. Ignored \begin{aligned} ... \end{aligned}]

)   (10)

Until now , which will be substituted into ISS internal multiple attenuator to predict internal multiple .

Based on Weglein et al. (2003), the 1D ISS internal multiple attenuation algorithm is

(11)

where ε is used to make sure the events satisfy lower-higher-lower relation, and its value is chosen based on the length of wavelet.

For this model, there are two primaries in the data. Now I suppose these two events are isolated (Figure 3). The pseudo depths of the first event is with a length of 2a, whereas the pseudo depth of the second event is with a length of

2b. For ε in eqn.11 , it is chosen to satisfy $\varepsilon \geq max(2a,2b)$ and .

Figure 3: data in pseudo depth domain with two primaries

Kaplan et al. (2004) change the integral order of eqn.11, and rewrite the formula as :

(12)

Since , eqn. 12 can be divided into two parts:

[Sorry. Ignored `\begin{aligned} ... \end{aligned}`]

(13)

For (13.1), the integral limitation of z is . Consider the lower limit of the integral of z' and the constrain of ε,

,

and

.

We can see the lower limit of second integral should be after the end of the first event and before the beginning of the second event, meaning in $[z+\varepsilon,\infty)$, the kernel of second integral .

So

(14)

Similarly, for (13.2), the integral limitation of z is . Consider the lower limit of the integral of z' and the constrain of ε,

.

The lower limit of second integral should be after the end of the second event, meaning in $[z+\varepsilon,\infty)$, the kernel of second integral .

So

(15)

Now

(16)

Since the actual first order internal multiple in domain is

(17)

The relation between predicted and actual multiple is

(18)

By using the ISS internal multiple algorithm, the multiple can be predicted with a right phase and approximated amplitude.

If the data is without Q influence, then from Weglein et al. (2003), we can get

(19)

Comparing eqn.18 and eqn.19, it's not difficult to know the predicted amplitude is worse for input data with Q relative to that without Q.

4 Numerical test on input data with Q

In this section, a two-reflector 1D model (Figure 2) will be used as an example to numerically test the ISS internal multiple attenuator. The parameters are listed in table 1.

Table 1: two-reflector model parameters

Layer Number	Velocity(m/s)	Density(kg/)	Travel Times (s)	Q Value
1	1500	1000	0.5	∞(5000)
2	4000	1000	1.1	100
3	2000	1000		

By using the parameters of table 1, based on eqn.5-7, eqn.9 and eqn.17, the existing 1D plane wave modeling code of M-OSRP is modified and the synthetic data involving Q value of each layer is generalized. The data includes all the primaries and the first order internal multiples.

Figure 4: Numerical test result. left: input data; middle: input data (blue line) and predicted multiple (red line); right: actual internal multiple (blue line) and predicted internal multiple (red line)

Substitute the input data (Figure 4, left blue line) into ISS internal multiple attenuation algorithm, we can get predict internal multiple (Figure 4 left red line). Actually the red line in Figure 4 left part is -, since we can see from eqn.18 that the polarity of is different with actual internal multiple. In order to more clearly show the result, the predicted multiple and actual multiple are compared separately, shown in Figure 4 right part. From the result, we can further realize the prediction result matches well in phase and approximately in amplitude.

5 Conclusion and future plan

In this paper, ISS internal multiple attenuator is tested analytically and numerically using Q influenced input data , with the conclusion that if events are not interfering seriously, the prediction will have a right phase and approximate amplitude. However, the prediction is worth than that without Q from analytical result. So Q compensation is important for ISS internal multiple attenuator even eliminator to have more effective and significant result.

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